## Piers and Columns

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Transportation
2.1 Introduction ..... 2-1
2.2 Structural Types ..... 2-1General • Selection Criteria
2.3 Design Loads ..... 2-4
Live Loads • Thermal Forces
2.4 Design Criteria ..... 2-7Overview • Slenderness and Second-Order Effect •Concrete Piers and Columns • Steel and CompositeColumns

### 2.1 Introduction

Piers provide vertical supports for spans at intermediate points and perform two main functions: transferring superstructure vertical loads to the foundations and resisting horizontal forces acting on the bridge. Although piers are traditionally designed to resist vertical loads, it is becoming more and more common to design piers to resist high lateral loads caused by seismic events. Even in some low seismic areas, designers are paying more attention to the ductility aspect of the design. Piers are predominantly constructed using reinforced concrete. Steel, to a lesser degree, is also used for piers. Steel tubes filled with concrete (composite) columns have gained more attention recently.

This chapter deals only with piers or columns for conventional bridges, such as grade separations, overcrossings, overheads, underpasses, and simple river crossings. Reinforced concrete columns will be discussed in detail while steel and composite columns will be briefly discussed. Substructures for arch, suspension, segmental, cable-stayed, and movable bridges are excluded from this chapter. Chapter 3 discusses the substructures for some of these special types of bridges.

### 2.2 Structural Types

### 2.2.1 General

Pier is usually used as a general term for any type of substructure located between horizontal spans and foundations. However, from time to time, it is also used particularly for a solid wall in order to distinguish it from columns or bents. From a structural point of view, a column is a member that resists the lateral force mainly by flexure action whereas a pier is a member that resists the lateral force mainly by a shear mechanism. A pier that consists of multiple columns is often called a bent.

There are several ways of defining pier types. One is by its structural connectivity to the superstructure: monolithic or cantilevered. Another is by its sectional shape: solid or hollow; round, octagonal, hexagonal, or rectangular. It can also be distinguished by its framing configuration: single or multiple column bent; hammerhead or pier wall.


FIGURE 2.1 Typical cross-section shapes of piers for overcrossings or viaducts on land.


FIGURE 2.2 Typical cross-section shapes of piers for river and waterway crossings.

### 2.2.2 Selection Criteria

Selection of the type of piers for a bridge should be based on functional, structural, and geometric requirements. Aesthetics is also a very important factor of selection since modern highway bridges are part of a city's landscape. Figure 2.1 shows a collection of typical cross section shapes for overcrossings and viaducts on land and Figure 2.2 shows some typical cross section shapes for piers of river and waterway crossings. Often, pier types are mandated by government agencies or owners. Many state departments of transportation in the United States have their own standard column shapes.

Solid wall piers, as shown in Figures 2.3a and 2.4, are often used at water crossings since they can be constructed to proportions that are both slender and streamlined. These features lend themselves well for providing minimal resistance to flood flows.

(a) Solid wall pier

(b) Hammerhead pier

(c) Rigid frame pier

FIGURE 2.3 Typical pier types for steel bridges.


FIGURE 2.4 Typical pier types and configurations for river and waterway crossings.
Hammerhead piers, as shown in Figure 2.3b, are often found in urban areas where space limitation is a concern. They are used to support steel girder or precast prestressed concrete superstructures. They are aesthetically appealing. They generally occupy less space, thereby providing more room for the traffic underneath. Standards for the use of hammerhead piers are often maintained by individual transportation departments.

A column bent pier consists of a cap beam and supporting columns forming a frame. Column bent piers, as shown in Figure 2.3c and Figure 2.5, can either be used to support a steel girder superstructure or be used as an integral pier where the cast-in-place construction technique is used. The columns can be either circular or rectangular in cross section. They are by far the most popular forms of piers in the modern highway system.

A pile extension pier consists of a drilled shaft as the foundation and the circular column extended from the shaft to form the substructure. An obvious advantage of this type of pier is that it occupies a minimal amount of space. Widening an existing bridge in some instances may require pile extensions because limited space precludes the use of other types of foundations.

(a) Bent for precast girders

(b) Bent for cast-in-place girders

FIGURE 2.5 Typical pier types for concrete bridges.
Selections of proper pier type depend upon many factors. First of all, it depends upon the type of superstructure. For example, steel girder superstructures are normally supported by cantilevered piers, whereas the cast-in-place concrete superstructures are normally supported by monolithic bents. Second, it depends upon whether the bridges are over a waterway or not. Pier walls are preferred on river crossings, where debris is a concern and hydraulics dictates it. Multiple pile extension bents are commonly used on slab bridges. Last, the height of piers also dictates the type selection of piers. The taller piers often require hollow cross sections in order to reduce the weight of the substructure. This then reduces the load demands on the costly foundations. Table 2.1 summarizes the general type selection guidelines for different types of bridges.

### 2.3 Design Loads

Piers are commonly subjected to forces and loads transmitted from the superstructure, and forces acting directly on the substructure. Some of the loads and forces to be resisted by the substructure include:

- Dead loads
- Live loads and impact from the superstructure
- Wind loads on the structure and the live loads
- Centrifugal force from the superstructure
- Longitudinal force from live loads
- Drag forces due to the friction at bearings
- Earth pressure
- Stream flow pressure
- Ice pressure
- Earthquake forces
- Thermal and shrinkage forces
- Ship impact forces
- Force due to prestressing of the superstructure
- Forces due to settlement of foundations

TABLE 2.1 General Guidelines for Selecting Pier Types

| Applicable Pier Types |  |  |
| :---: | :---: | :---: |
| Steel Superstructure |  |  |
| Over water | Tall piers | Pier walls or hammerheads (T-piers) (Figures 2.3a and b); hollow cross sections for most cases; cantilevered; could use combined hammerheads with pier wall base and step tapered shaft |
|  | Short piers | Pier walls or hammerheads (T-piers) (Figures 2.3a and b); solid cross sections; cantilevered |
| On land | Tall piers | Hammerheads (T-piers) and possibly rigid frames (multiple column bents)(Figures 2.3b and c); hollow cross sections for single shaft and solid cross sections for rigid frames; cantilevered |
|  | Short piers | Hammerheads and rigid frames (Figures 2.3b and c); solid cross sections; cantilevered |
| Precast Prestressed Concrete Superstructure |  |  |
| Over water | Tall piers | Pier walls or hammerheads (Figure 2.4); hollow cross sections for most cases; cantilevered; could use combined hammerheads with pier wall base and step-tapered shaft |
|  | Short piers | Pier walls or hammerheads; solid cross sections; cantilevered |
| On land | Tall piers | Hammerheads and possibly rigid frames (multiple column bents); hollow cross sections for single shafts and solid cross sections for rigid frames; cantilevered |
|  | Short piers | Hammerheads and rigid frames (multiple column bents) (Figure 2.5a); solid cross sections; cantilevered |
| Cast-in-Place Concrete Superstructure |  |  |
| Over water | Tall piers | Single shaft pier (Figure 2.4); superstructure will likely cast by traveled forms with balanced cantilevered construction method; hollow cross sections; monolithic; fixed at bottom |
|  | Short piers | Pier walls (Figure 2.4); solid cross sections; monolithic; fixed at bottom |
| On land | Tall piers Short piers | Single or multiple column bents; solid cross sections for most cases, monolithic; fixed at bottom Single or multiple column bents (Figure 2.5b); solid cross sections; monolithic; pinned at bottom |

The effect of temperature changes and shrinkage of the superstructure needs to be considered when the superstructure is rigidly connected with the supports. Where expansion bearings are used, forces caused by temperature changes are limited to the frictional resistance of bearings.

In the following, two load cases, live loads and thermal forces, will be discussed in detail because they are two of the most common loads on the piers, but are often applied incorrectly.

### 2.3.1 Live Loads

Bridge live loads are the loads specified or approved by the contracting agencies and owners. They are usually specified in the design codes such as AASHTO LRFD Bridge Design Specifications [1]. There are other special loading conditions peculiar to the type or location of the bridge structure which should be specified in the contracting documents.

Live-load reactions obtained from the design of individual members of the superstructure should not be used directly for substructure design. These reactions are based upon maximum conditions for one beam and make no allowance for distribution of live loads across the roadway. Use of these maximum loadings would result in a pier design with an unrealistically severe loading condition and uneconomical sections.

For substructure design, a maximum design traffic lane reaction using either the standard truck load or standard lane load should be used. Design traffic lanes are determined according to AASHTO LRFD [1] Section 3.6. For the calculation of the actual beam reactions on the piers, the maximum lane reaction can be applied within the design traffic lanes as wheel loads, and then distributed to the beams assuming the slab between beams to be simply supported (Figure 2.6). Wheel loads can be positioned anywhere within the design traffic lane with a minimum distance between lane boundary and wheel load of $0.61 \mathrm{~m}(2 \mathrm{ft})$.


* DESIGN TRAFFIC LANE $=3.6 \mathrm{~m}$

WHEEL LOADING $W=\frac{R_{2}}{2}$ NO. OF LANES $=$ ROADWAY $\div 3.6$ REDUCED TO NEAREST WHOLE NUMBER


FIGURE 2.6 Wheel load arrangement to produce maximum positive moment.

The design traffic lanes and the live load within the lanes should be arranged to produce beam reactions that result in maximum loads on the piers. AASHTO LRFD Section 3.6.1.1.2 provides load reduction factors due to multiple loaded lanes.

Live-load reactions will be increased due to impact effect. AASHTO LRFD [1] refers to this as the dynamic load allowance, IM. and is listed here as in Table 2.2.

TABLE 2.2 Dynamic Load Allowance, IM

| Component | IM |
| :---: | :---: |
| Deck joints - all limit states | 75\% |
| All other components |  |
| - Fatigue and fracture limit state | 15\% |
| - All other limit states | 33\% |

### 2.3.2 Thermal Forces

Forces on piers due to thermal movements, shrinkage, and prestressing can become large on short, stiff bents of prestressed concrete bridges with integral bents. Piers should be checked against these forces. Design codes or specifications normally specify the design temperature range. Some codes even specify temperature distribution along the depth of the superstructure member.

The first step in determining the thermal forces on the substructures for a bridge with integral bents is to determine the point of no movement. After this point is determined, the relative displacement of any point along the superstructure to this point is simply equal to the distance to this point times the temperature range and times the coefficient of expansion. With known displacement at the top and known boundary conditions at the top and bottom, the forces on the pier due to the temperature change can be calculated by using the displacement times the stiffness of the pier.

The determination of the point of no movement is best demonstrated by the following example, which is adopted from Memo to Designers issued by California Department of Transportation [2]:

## Example 2.1

A $225.55-\mathrm{m}$ ( 740 -foot)-long and $23.77-\mathrm{m}$ ( 78 -foot) wide concrete box-girder superstructure is supported by five two-column bents. The size of the column is $1.52 \mathrm{~m}(5 \mathrm{ft})$ in diameter and the heights vary between $10.67 \mathrm{~m}(35 \mathrm{ft})$ and $12.80 \mathrm{~m}(42 \mathrm{ft})$. Other assumptions are listed in the calculations. The calculation is done through a table. Please refer Figure 2.7 for the calculation for determining the point of no movement.

### 2.4 Design Criteria

### 2.4.1 Overview

Like the design of any structural component, the design of a pier or column is performed to fulfill strength and serviceability requirements. A pier should be designed to withstand the overturning, sliding forces applied from superstructure as well as the forces applied to substructures. It also needs to be designed so that during an extreme event it will prevent the collapse of the structure but may sustain some damage.

A pier as a structure component is subjected to combined forces of axial, bending, and shear. For a pier, the bending strength is dependent upon the axial force. In the plastic hinge zone of a pier, the shear strength is also influenced by bending. To complicate the behavior even more, the bending moment will be magnified by the axial force due to the $P-\Delta$ effect.

In current design practice, the bridge designers are becoming increasingly aware of the adverse effects of earthquake. Therefore, ductility consideration has become a very important factor for bridge design. Failure due to scouring is also a common cause of failure of bridges. In order to prevent this type of failure, the bridge designers need to work closely with the hydraulic engineers to determine adequate depths for the piers and provide proper protection measures.

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### 2.4.2 Slenderness and Second-Order Effect

The design of compression members must be based on forces and moments determined from an analysis of the structure. Small deflection theory is usually adequate for the analysis of beam-type members. For compression members, however, the second-order effect must be considered. According to AASHTO LRFD [1], the second-order effect is defined as follows:

The presence of compressive axial forces amplify both out-of-straightness of a component and the deformation due to non-tangential loads acting thereon, therefore increasing the eccentricity of the axial force with respect to the centerline of the component. The synergistic effect of this interaction is the apparent softening of the component, i.e., a loss of stiffness.

To assess this effect accurately, a properly formulated large deflection nonlinear analysis can be performed. Discussions on this subject can be found in References [3,4]. However, it is impractical to expect practicing engineers to perform this type of sophisticated analysis on a regular basis. The moment magnification procedure given in AASHTO LRFD [1] is an approximate process which was selected as a compromise between accuracy and ease of use. Therefore, the AASHTO LRFD moment magnification procedure is outlined in the following.

When the cross section dimensions of a compression member are small in comparison to its length, the member is said to be slender. Whether or not a member can be considered slender is dependent on the magnitude of the slenderness ratio of the member. The slenderness ratio of a compression member is defined as, $K L_{u} / r$, where $K$ is the effective length factor for compression members; $L_{u}$ is the unsupported length of compression member; $r$ is the radius of gyration $=\sqrt{I / A}$; $I$ is the moment of inertia; and $A$ is the cross-sectional area.

When a compression member is braced against side sway, the effective length factor, $K=1.0$ can be used. However, a lower value of $K$ can be used if further analysis demonstrates that a lower value is applicable. $L_{u}$ is defined as the clear distance between slabs, girders, or other members which is capable of providing lateral support for the compression member. If haunches are present, then, the unsupported length is taken from the lower extremity of the haunch in the plane considered (AASHTO LRFD 5.7.4.3). For a detailed discussion of the $K$-factor, please refer to Chapter 8.

For a compression member braced against side sway, the effects of slenderness can be ignored as long as the following condition is met (AASHTO LRFD 5.7.4.3):

$$
\begin{equation*}
\frac{K L_{u}}{r}<34-\left(\frac{12 M_{1 b}}{M_{2 b}}\right) \tag{2.1}
\end{equation*}
$$

where
$M_{1 b}=$ smaller end moment on compression member - positive if member is bent in single curvature, negative if member is bent in double curvature
$M_{2 b}=$ larger end moment on compression member - always positive
For an unbraced compression member, the effects of slenderness can be ignored as long as the following condition is met (AASHTO LRFD 5.7.4.3):

$$
\begin{equation*}
\frac{K L_{u}}{r}<22 \tag{2.2}
\end{equation*}
$$

If the slenderness ratio exceeds the above-specified limits, the effects can be approximated through the use of the moment magnification method. If the slenderness ratio $K L_{u} / r$ exceeds 100 , however, a more-detailed second-order nonlinear analysis will be required. Any detailed analysis should consider the influence of axial loads and variable moment of inertia on member stiffness and forces, and the effects of the duration of the loads.

The factored moments may be increased to reflect effects of deformations as follows:

$$
\begin{equation*}
M_{c}=\delta_{b} M_{2 b}+\delta_{s} M_{2 s} \tag{2.3}
\end{equation*}
$$

where
$M_{2 b}=$ moment on compression member due to factored gravity loads that result in no appreciable side sway calculated by conventional first-order elastic frame analysis, always positive
$M_{2 s}=$ moment on compression member due to lateral or gravity loads that result in side sway, $\Delta$, greater than $L_{u} / 1500$, calculated by conventional first-order elastic frame analysis, always positive
The moment magnification factors are defined as follows:

$$
\begin{align*}
& \delta_{b}=\frac{C_{m}}{1-\frac{P_{u}}{\phi P_{c}}} \geq 1.0  \tag{2.4}\\
& \delta_{s}=\frac{1}{1-\frac{\sum P_{u}}{\phi \sum P_{c}}} \geq 1.0 \tag{2.5}
\end{align*}
$$

where
$P_{u}=$ factored axial load
$P_{c}=$ Euler buckling load, which is determined as follows:

$$
\begin{equation*}
P_{c}=\frac{\pi^{2} E I}{\left(K L_{u}\right)^{2}} \tag{2.6}
\end{equation*}
$$

$C_{m}$, a factor which relates the actual moment diagram to an equivalent uniform moment diagram, is typically taken as 1.0 . However, in the case where the member is braced against side sway and without transverse loads between supports, it may be taken by the following expression:

$$
\begin{equation*}
C_{m}=0.60+0.40\left(\frac{M_{1 b}}{M_{2 b}}\right) \tag{2.7}
\end{equation*}
$$

The value resulting from Eq. (2.7), however, is not to be less than 0.40 .
To compute the flexural rigidity $E I$ for concrete columns, AASHTO offers two possible solutions, with the first being:

$$
\begin{equation*}
E I=\frac{\frac{E_{c} I_{g}}{5}+E_{s} I_{s}}{1+\beta_{d}} \tag{2.8}
\end{equation*}
$$

and the second, more-conservative solution being:

$$
\begin{equation*}
E I=\frac{\frac{E_{c} I_{g}}{2.5}}{1+\beta_{d}} \tag{2.9}
\end{equation*}
$$

where $E_{c}$ is the elastic modulus of concrete, $I_{g}$ is the gross moment inertia, $E_{s}$ is the elastic modules of reinforcement, $I_{s}$ is the moment inertia of reinforcement about centroidal axis, and $\beta$ is the ratio of maximum dead-load moment to maximum total-load moment and is always positive. It is an approximation of the effects of creep, so that when larger moments are induced by loads sustained over a long period of time, the creep deformation and associated curvature will also be increased.

### 2.4.3 Concrete Piers and Columns

### 2.4.3.1 Combined Axial and Flexural Strength

A critical aspect of the design of bridge piers is the design of compression members. We will use AASHTO LRFD Bridge Design Specifications [1] as the reference source. The following discussion provides an overview of some of the major criteria governing the design of compression members.

Under the Strength Limit State Design, the factored resistance is determined with the product of nominal resistance, $P_{n}$, and the resistance factor, $\phi$. Two different values of $\phi$ are used for the nominal resistance $P_{n}$. Thus, the factored axial load resistance $\phi P_{n}$ is obtained using $\phi=0.75$ for columns with spiral and tie confinement reinforcement. The specifications also allows for the value $\phi$ to be linearly increased from the value stipulated for compression members to the value specified for flexure which is equal to 0.9 as the design axial load $\phi P_{n}$ decreases from $0.10 f_{c}^{\prime} A_{g}$ to zero.

## Interaction Diagrams

Flexural resistance of a concrete member is dependent upon the axial force acting on the member. Interaction diagrams are usually used as aids for the design of the compression members. Interaction diagrams for columns are usually created assuming a series of strain distributions, and computing the corresponding values of $P$ and $M$. Once enough points have been computed, the results are plotted to produce an interaction diagram.

Figure 2.8 shows a series of strain distributions and the resulting points on the interaction diagram. In an actual design, however, a few points on the diagrams can be easily obtained and can define the diagram rather closely.

- Pure Compression:

The factored axial resistance for pure compression, $\phi P_{n}$, may be computed by:
For members with spiral reinforcement:

$$
\begin{equation*}
P_{r}=\phi P_{n}=\phi 0.85 P_{o}=\phi 0.85\left[0.85 f_{c}^{\prime}\left(A_{g}-A_{s t}\right)+A_{s t} f_{y}\right] \tag{2.10}
\end{equation*}
$$

For members with tie reinforcement:

$$
\begin{equation*}
P_{r}=\phi P_{n}=\phi 0.80 P_{o}=\phi 0.80\left[0.85 f_{c}^{\prime}\left(A_{g}-A_{s t}\right)+A_{s t} f_{y}\right] \tag{2.11}
\end{equation*}
$$

For design, pure compression strength is a hypothetical condition since almost always there will be moments present due to various reasons. For this reason, AASHTO LRFD 5.7.4.4 limits the nominal axial load resistance of compression members to 85 and $80 \%$ of the axial resistance at zero eccentricity, $P_{o}$, for spiral and tied columns, respectively.

- Pure Flexure:

The section in this case is only subjected to bending moment and without any axial force. The factored flexural resistance, $M_{r}$, may be computed by


FIGURE 2.8 Strain distributions corresponding to points on interaction diagram.

$$
\begin{align*}
M_{r} & =\phi M_{n}=\phi\left[A_{s} f_{y} d\left(1-0.6 \rho \frac{f_{y}}{f_{c}^{\prime}}\right)\right]  \tag{2.12}\\
& =\phi\left[A_{s} f_{y}\left(d-\frac{a}{2}\right)\right]
\end{align*}
$$

where

$$
a=\frac{A_{s} f_{y}}{0.85 f_{c}^{\prime} b}
$$

- Balanced Strain Conditions:

Balanced strain conditions correspond to the strain distribution where the extreme concrete strain reaches 0.003 and the strain in reinforcement reaches yield at the same time. At this condition, the section has the highest moment capacity. For a rectangular section with reinforcement in one face, or located in two faces at approximately the same distance from the axis of bending, the balanced factored axial resistance, $P_{r}$, and balanced factored flexural resistance, $M_{r}$, may be computed by

$$
\begin{equation*}
P_{r}=\phi P_{b}=\phi\left[0.85 f_{c}^{\prime} b a_{b}+A_{s}^{\prime} f_{s}^{\prime}-A_{s} f_{y}\right] \tag{2.13}
\end{equation*}
$$

and

$$
\begin{equation*}
M_{r}=\phi M_{b}=\phi\left[0.85 f_{c}^{\prime} b a_{b}\left(d-d^{\prime \prime}-a_{b} / 2\right)+A_{s}^{\prime} f_{s}^{\prime}\left(d-d^{\prime}-d^{\prime \prime}\right)+A_{s} f_{y} d^{\prime \prime}\right] \tag{2.14}
\end{equation*}
$$

where

$$
a_{b}=\left(\frac{600}{600+f_{y}}\right) \beta_{1} d
$$

and

$$
f_{s}^{\prime}=600\left[1-\left(\frac{d^{\prime}}{d}\right)\left(600+\frac{f_{y}}{600}\right)\right] \leq f_{y}
$$

where $f_{y}$ is in MPa.

## Biaxial Bending

AASHTO LRFD 5.7.4.5 stipulates that the design strength of noncircular members subjected to biaxial bending may be computed, in lieu of a general section analysis based on stress and strain compatibility, by one of the following approximate expressions:

$$
\begin{equation*}
\frac{1}{P_{r x y}}=\frac{1}{P_{r x}}+\frac{1}{P_{r y}}-\frac{1}{P_{o}} \tag{2.15}
\end{equation*}
$$

when the factored axial load, $P_{u} \square 0.10 \phi f_{c}^{\prime} A_{g}$

$$
\begin{equation*}
\frac{M_{u x}}{M_{r x}}+\frac{M_{u y}}{M_{r y}} \leq 1 \tag{2.16}
\end{equation*}
$$

when the factored axial load, $P_{u}<0.10 \phi f_{c}^{\prime} A_{g}$
where
$P_{r x y} \quad=$ factored axial resistance in biaxial flexure
$P_{r x}, P_{r y}=$ factored axial resistance corresponding to $M_{r x}, M_{r y}$
$M_{u x}, M_{u y}=$ factored applied moment about the $x$-axis, $y$-axis
$M_{r x}, M_{r y}=$ uniaxial factored flexural resistance of a section about the $x$-axis and $y$-axis corresponding to the eccentricity produced by the applied factored axial load and moment, and
$P_{o} \quad=0.85 f_{c}^{\prime}\left(A_{g}-A_{s}\right)+A_{s} f_{y}$

### 2.4.3.2 Shear Strength

Under the normal load conditions, the shear seldom governs the design of the column for conventional bridges since the lateral loads are usually small compared with the vertical loads. However, in a seismic design, the shear is very important. In recent years, the research effort on shear strength evaluation for columns has been increased remarkably. AASHTO LRFD provides a general shear equation that applies for both beams and columns. The concrete shear capacity component and the angle of inclination of diagonal compressive stresses are functions of the shear stress on the concrete and the strain in the reinforcement on the flexural tension side of the member. It is rather involved and hard to use.

Alternatively, the equations recommended by ATC-32 [5] can be used with acceptable accuracy. The recommendations are listed as follows.

Except for the end regions of ductile columns, the nominal shear strength provided by concrete, $V_{c}$, for members subjected to flexure and axial compression should be computed by

$$
\begin{equation*}
V_{c}=0.165\left(1+(3.45)\left(10^{-6}\right) \frac{N_{u}}{A_{g}}\right) \sqrt{f_{c}^{\prime} A_{e}} \quad(\mathrm{MPa}) \tag{2.17}
\end{equation*}
$$

If the axial force is in tension, the $V_{c}$ should be computed by

$$
\begin{equation*}
V_{c}=0.165\left(1+(1.38)\left(10^{-5}\right) \frac{N_{u}}{A_{g}}\right) \sqrt{f_{c}^{\prime}} A_{e} \quad(\mathrm{MPa}) \tag{2.18}
\end{equation*}
$$

(note that $N_{u}$ is negative for tension),
where
$A_{g}=$ gross section area of the column $\left(\mathrm{mm}^{2}\right)$
$A_{e}=$ effective section area, can be taken as $0.8 A_{g}\left(\mathrm{~mm}^{2}\right)$
$N_{u}=$ axial force applied to the column (N)
$f_{c}^{\prime}=$ compressive strength of concrete (MPa)
For end regions where the flexural ductility is normally high, the shear capacity should be reduced. ATC-32 [5] offers the following equations to address this interaction.

With the end region of columns extending a distance from the critical section or sections not less than $1.5 D$ for circular columns or $1.5 h$ for rectangular columns, the nominal shear strength provided by concrete subjected to flexure and axial compression should be computed by

$$
\begin{equation*}
V_{c}=0.165\left(0.5+(6.9)\left(10^{-6}\right) \frac{N_{u}}{A_{g}}\right) \sqrt{f_{c}^{\prime} A_{e}} \quad(\mathrm{MPa}) \tag{2.19}
\end{equation*}
$$

When axial load is tension, $V_{c}$ can be calculated as

$$
\begin{equation*}
V_{c}=0.165\left(1+(1.38)\left(10^{-5}\right) \frac{N_{u}}{A_{g}}\right) \sqrt{f_{c}^{\prime}} A_{e} \quad(\mathrm{MPa}) \tag{2.18}
\end{equation*}
$$

Again, $N_{u}$ should be negative in this case.
The nominal shear contribution from reinforcement is given by

$$
\begin{equation*}
V_{s}=\frac{A_{v} f_{y h} d}{s} \quad(\mathrm{MPa}) \tag{2.20}
\end{equation*}
$$

for tied rectangular sections, and by

$$
\begin{equation*}
V_{s}=\frac{\pi}{2} \frac{A_{h} f_{y h} D^{\prime}}{s} \tag{2.21}
\end{equation*}
$$

for spirally reinforced circular sections. In these equations, $A_{\nu}$ is the total area of shear reinforcement parallel to the applied shear force, $A_{h}$ is the area of a single hoop, $f_{y h}$ is the yield stress of horizontal reinforcement, $D^{\prime}$ is the diameter of a circular hoop, and $s$ is the spacing of horizontal reinforcement.

### 2.4.3.3 Ductility of Columns

The AASHTO LRFD [1] introduces the term ductility and requires that a structural system of bridge be designed to ensure the development of significant and visible inelastic deformations prior to failure.

The term ductility defines the ability of a structure and selected structural components to deform beyond elastic limits without excessive strength or stiffness degradation. In mathematical terms, the ductility $\mu$ is defined by the ratio of the total imposed displacement $\Delta$ at any instant to that at the onset of yield $\Delta_{y}$. This is a measure of the ability for a structure, or a component of a structure, to absorb energy. The goal of seismic design is to limit the estimated maximum ductility demand to the ductility capacity of the structure during a seismic event.

For concrete columns, the confinement of concrete must be provided to ensure a ductile column. AASHTO LRFD [1] specifies the following minimum ratio of spiral reinforcement to total volume of concrete core, measured out-to-out of spirals:

$$
\begin{equation*}
\rho_{s}=0.45\left(\frac{A_{g}}{A_{c}}-1\right) \frac{f_{c}^{\prime}}{f_{y h}} \tag{2.22}
\end{equation*}
$$

The transverse reinforcement for confinement at the plastic hinges shall be determined as follows:

$$
\begin{equation*}
\rho_{s}=0.16 \frac{f_{c}^{\prime}}{f_{y}}\left(0.5+\frac{1.25 P_{u}}{A_{g} f_{c}^{\prime}}\right) \tag{2.23}
\end{equation*}
$$

for which

$$
\left(0.5+\frac{1.25 P_{u}}{A_{g} f_{c}^{\prime}}\right) \geq 1.0
$$

The total cross-sectional area $\left(A_{s h}\right)$ of rectangular hoop (stirrup) reinforcement for a rectangular column shall be either

$$
\begin{equation*}
A_{s h}=0.30 a h_{c} \frac{f_{c}^{\prime}}{f_{y h}}\left(\frac{A_{g}}{A_{c}}-1\right) \tag{2.24}
\end{equation*}
$$

or,

$$
\begin{equation*}
A_{s h}=0.12 a h_{c} \frac{f_{c}^{\prime}}{f_{y}}\left(0.5+\frac{1.25 P_{u}}{A_{g} f_{c}^{\prime}}\right) \tag{2.25}
\end{equation*}
$$

whichever is greater,
where
$a=$ vertical spacing of hoops (stirrups) with a maximum of $100 \mathrm{~mm}(\mathrm{~mm})$
$A_{c}=$ area of column core measured to the outside of the transverse spiral reinforcement $\left(\mathrm{mm}^{2}\right)$
$A_{g}=$ gross area of column ( $\mathrm{mm}^{2}$ )
$A_{s h}=$ total cross-sectional area of hoop (stirrup) reinforcement ( $\mathrm{mm}^{2}$ )
$f_{c}^{\prime}=$ specified compressive strength of concrete ( Pa )
$f_{y h}=$ yield strength of hoop or spiral reinforcement ( Pa )
$h_{c}=$ core dimension of tied column in the direction under consideration (mm)
$\rho_{s}=$ ratio of volume of spiral reinforcement to total volume of concrete core (out-to-out of spiral)
$P_{u}=$ factored axial load (MN)


FIGURE 2.9 Example 2.2 - typical section.

TABLE 2.3 Column Group Loads - Service

|  | Dead Load | Live Load + Impact |  |  | Win <br> d | Wind on LL | Long <br> Force | Centrifugal Force- $M_{y}$ | Temp. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Case 1 | Case 2 | Case 3 |  |  |  |  |  |
|  |  | Trans $M_{y-\text { max }}$ | Long $M_{x-\text { max }}$ | Axial N-max |  |  |  |  |  |
| $M_{y}(\mathrm{k}-\mathrm{ft})$ | 220 | 75 | 15 | 32 | 532 | 153 | 208 | 127 | 180 |
| $M_{x}(\mathrm{k}-\mathrm{ft})$ | 148 | 67 | 599 | 131 | 192 | 86 | 295 | 2 | 0 |
| $P(\mathrm{k})$ | 1108 | 173 | 131 | 280 | 44 | 17 | 12 | 23 | 0 |

TABLE 2.4 Unreduced Seismic Loads (ARS)

|  | Case 1 <br> Max. Transverse | Case 2 <br> Max. Longitudinal |
| :--- | :---: | :---: |
| $M_{y}$ — Trans (k-ft) | 4855 | 3286 |
| $M_{x}$ — Long (k-ft) | 3126 | 3334 |
| P—Axial (k) | -282 | -220 |

## Example 2.2 Design of a Two-Column Bent

Design the columns of a two-span overcrossing. The typical section of the structure is shown in Figure 2.9. The concrete box girder is supported by a two-column bent and is subjected to HS20 loading. The columns are pinned at the bottom of the columns. Therefore, only the loads at the top of columns are given here. Table 2.3 lists all the forces due to live load plus impact. Table 2.4 lists the forces due to seismic loads. Note that a load reduction factor of 5.0 will be assumed for the columns.

Material Data
$f_{c}^{\prime}=4.0 \mathrm{ksi}(27.6 \mathrm{MPa}) \quad E_{c}=3605 \mathrm{ksi}(24855 \mathrm{MPa})$
$E_{s}=29000 \mathrm{ksi}(199946 \mathrm{MPa}) \quad f_{y}=60 \mathrm{ksi}(414 \mathrm{MPa})$
Try a column size of $4 \mathrm{ft}(1.22 \mathrm{~m})$ in diameter. Provide 26-\#9 (26-\#30) longitudinal reinforcement. The reinforcement ratio is $1.44 \%$.


FIGURE 2.10 Example 2.2 - interaction diagram.

Section Properties
$A_{g}=12.51 \mathrm{ft}^{2}\left(1.16 \mathrm{~m}^{2}\right)$
$A_{s t}=26.0 \mathrm{in}^{2}\left(16774 \mathrm{~mm}^{2}\right)$
$I_{x c}=I_{y c}=12.46 \mathrm{ft}^{4}\left(0.1075 \mathrm{~m}^{4}\right)$
$I_{x s}=I_{y s}=0.2712 \mathrm{ft}^{4}\left(0.0023 \mathrm{~m}^{4}\right)$

The analysis follows the procedure discussed in Section 2.4.3.1. The moment and axial force interaction diagram is generated and is shown in Figure 2.10.

Following the procedure outlined in Section 2.4.2, the moment magnification factors for each load group can be calculated and the results are shown in Table 2.5.

In which:

$$
\begin{gathered}
K_{y}=K_{x}=2.10 \\
K_{y} L / R=K_{x} L / R=2.1 \times 27.0 /(1.0)=57
\end{gathered}
$$

where $R=$ radio of gyration $=r / 2$ for a circular section.

$$
22<K L / R<100 \quad \therefore \text { Second-order effect should be considered. }
$$

TABLE 2.5 Moment Magnification and Buckling Calculations

| Load <br> Group | $P(\mathrm{k})$ <br> Case | Moment Magnification |  |  | Cracked Transformed Section |  | Critical Buckling |  | Axial <br> Load <br> $P(\mathrm{k})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Trans. $M_{a g y}$ | Long $M_{\text {agx }}$ | Comb. $M_{a g}$ | $E^{*} I_{y}\left(\mathrm{k}-\mathrm{ft}^{2}\right)$ | $E^{\star} I_{x}\left(\mathrm{k}-\mathrm{ft}^{2}\right)$ | Trans. $P_{c y}(\mathrm{k})$ | Long $P_{c x}(\mathrm{k})$ |  |
| I | 1 | 1.571 | 1.640 | 1.587 | 1,738,699 | 1,619,399 | 5338 | 4972 | 1455 |
| I | 2 | 1.661 | 1.367 | 1.384 | 1,488,966 | 2,205,948 | 4571 | 6772 | 1364 |
| I | 3 | 2.765 | 2.059 | 2.364 | 1,392,713 | 1,728,396 | 4276 | 5306 | 2047 |
| II |  | 1.337 | 1.385 | 1.344 | 1,962,171 | 1,776,045 | 6024 | 5452 | 1137 |
| III | 1 | 1.406 | 1.403 | 1.405 | 2,046,281 | 2,056,470 | 6282 | 6313 | 1360 |
| III | 2 | 1.396 | 1.344 | 1.361 | 1,999,624 | 2,212,829 | 6139 | 6793 | 1305 |
| III | 3 | 1.738 | 1.671 | 1.708 | 1,901,005 | 2,011,763 | 5836 | 6176 | 1859 |
| IV | 1 | 1.437 | 1.611 | 1.455 | 1,864,312 | 1,494,630 | 5723 | 4588 | 1306 |
| IV | 2 | 1.448 | 1.349 | 1.377 | 1,755,985 | 2,098,586 | 5391 | 6443 | 1251 |
| IV | 3 | 1.920 | 1.978 | 1.936 | 1,635,757 | 1,585,579 | 5022 | 4868 | 1805 |
| V |  | 1.303 | 1.365 | 1.310 | 2,042,411 | 1,776,045 | 6270 | 5452 | 1094 |
| VI | 1 | 1.370 | 1.382 | 1.373 | 2,101,830 | 2,056,470 | 6453 | 6313 | 1308 |
| VI | 2 | 1.358 | 1.327 | 1.340 | 2,068,404 | 2,212,829 | 6350 | 6793 | 1256 |
| VI | 3 | 1.645 | 1.629 | 1.640 | 1,980,146 | 2,011,763 | 6079 | 6176 | 1788 |
| VII | 1 | 1.243 | 1.245 | 1.244 | 2,048,312 | 2,036,805 | 6288 | 6253 | 826 |
| VII | 2 | 1.296 | 1.275 | 1.286 | 1,940,100 | 2,053,651 | 5956 | 6305 | 888 |

Note: Column assumed to be unbraced against side sway.

The calculations for Loading Group III and Case 2 will be demonstrated in the following:
Bending in the longitudinal direction: $M_{x}$

$$
\text { Factored load }=1.3\left[\beta_{D} D+(L+I)+C F+0.3 W+W L+L F\right]
$$

$\beta_{D}=0.75$ when checking columns for maximum moment or maximum eccentricities and associated axial load. $\beta_{d}$ in Eq. (2.8) $=\max$ dead-load moment, $M_{\mathrm{DL}} / \max$ total moment, $M_{t}$.

$$
\begin{aligned}
M_{D L} & =148 \times 0.75=111 \mathrm{k}-\mathrm{ft}(151 \mathrm{kN} \cdot \mathrm{~m}) \\
M_{t} & =0.75 \times 148+599+0.3 \times 192+86+295+2=1151 \mathrm{k}-\mathrm{ft}(1561 \mathrm{kN} \cdot \mathrm{~m}) \\
\beta_{d} & =111 / 1151=0.0964 \\
E I_{x} & =\frac{\frac{E_{c} I_{g}}{5}+E_{s} I_{s}}{1+\beta_{d}}=\frac{3605 \times 144 \times 12.46}{5}+29,000 \times 144 \times 0.2712 \\
1+0.0964 & =2,212,829 \mathrm{k}-\mathrm{ft}^{2} \\
P_{c x} & =\frac{\pi^{2} E I_{x}}{\left(K L_{u}\right)^{2}}=\frac{\pi^{2} \times 2,212,829}{(2.1 \times 27)^{2}}=6793 \mathrm{kips}(30,229 \mathrm{kN}) \\
C_{m} & =1.0 \text { for frame braced against side sway }
\end{aligned}
$$

$$
\delta_{s}=\frac{1}{1-\frac{\sum P_{u}}{\phi \sum P_{c}}}=\frac{1}{1-\frac{1305}{0.75 \times 6793}}=1.344
$$

The magnified factored moment $=1.344 \times 1.3 \times 1151=2011 \mathrm{k}-\mathrm{ft}(2728 \mathrm{kN} \cdot \mathrm{m})$

TABLE 2.6 Comparison of Factored Loads to Factored Capacity of the Column

| Group | Case | Applied Factored Forces (k-ft) |  |  |  | Capacity (k-ft) |  | Ratio $M_{u} / M$ | Status |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Trans. $M_{y}$ | Long $M_{x}$ | Comb. M | Axial $P(\mathrm{k})$ | $\phi M_{n}$ | $\phi$ |  |  |
| I | 1 | 852 | 475 | 975 | 1455 | 2924 | 0.75 | 3.00 | OK |
| I | 2 | 566 | 1972 | 2051 | 1364 | 2889 | 0.75 | 1.41 | OK |
| I | 3 | 1065 | 981 | 1448 | 2047 | 3029 | 0.75 | 2.09 | OK |
| II |  | 1211 | 546 | 1328 | 1137 | 2780 | 0.75 | 2.09 | OK |
| III | 1 | 1622 | 1125 | 1974 | 1360 | 2886 | 0.75 | 1.46 | OK |
| III | 2 | 1402 | 2011 | 2449 | 1305 | 2861 | 0.75 | 1.17 | OK |
| III | 3 | 1798 | 1558 | 2379 | 1859 | 3018 | 0.75 | 1.27 | OK |
| IV | 1 | 1022 | 373 | 1088 | 1306 | 2865 | 0.75 | 2.63 | OK |
| IV | 2 | 813 | 1245 | 1487 | 1251 | 2837 | 0.75 | 1.91 | OK |
| IV | 3 | 1136 | 717 | 1343 | 1805 | 3012 | 0.75 | 2.24 | OK |
| V |  | 1429 | 517 | 1519 | 1094 | 2754 | 0.75 | 1.81 | OK |
| VI | 1 | 1829 | 1065 | 2116 | 1308 | 2864 | 0.75 | 1.35 | OK |
| VI | 2 | 1617 | 1905 | 2499 | 1256 | 2842 | 0.75 | 1.14 | OK |
| VI | 3 | 2007 | 1461 | 2482 | 1788 | 3008 | 0.75 | 1.21 | OK |
| VII | 1 | 1481 | 963 | 1766 | 826 | 2372 | 0.67 | 1.34 | OK |
| VII | 2 | 1136 | 1039 | 1540 | 888 | 2364 | 0.65 | 1.54 | OK |

Notes:

1. Applied factored moments are magnified for slenderness in accordance with AASHTO LRFD.
2. The seismic forces are reduced by the load reduction factor $R=5.0$.
$L=27.00 \mathrm{ft}, f_{c}^{\prime}=4.00 \mathrm{ksi}, F_{y}=60.0 \mathrm{ksi}, A_{s t}=26.00 \mathrm{in} .^{2}$

The analysis results with the comparison of applied moments to capacities are summarized in Table 2.6.

Column lateral reinforcement is calculated for two cases: (1) for applied shear and (2) for confinement. Typically, the confinement requirement governs. Apply Eq. 2.22 or Eq. 2.23 to calculate the confinement reinforcement. For seismic analysis, the unreduced seismic shear forces should be compared with the shear forces due to plastic hinging of columns. The smaller should be used. The plastic hinging analysis procedure is discussed elsewhere in this handbook and will not be repeated here.

The lateral reinforcement for both columns are shown as follows.

## For left column:

$$
\begin{aligned}
V_{u} & =148 \text { kips }(659 \mathrm{kN})(\text { shear due to plastic hinging governs }) \\
\phi V_{n} & =167 \mathrm{kips}(743 \mathrm{kN}) \quad \therefore \text { No lateral reinforcement is required for shear. }
\end{aligned}
$$

Reinforcement for confinement $=\rho_{s}=0.0057 \therefore$ Provide \#4 at 3 in. (\#15 at 76 mm )
For right column:

$$
\begin{aligned}
& V_{u}=180 \text { kips }(801 \mathrm{kN}) \text { (shear due to plastic hinging governs) } \\
& \phi V_{n}=167 \mathrm{kips}(734 \mathrm{kN}) \\
& \phi V_{s}=13 \mathrm{kips}(58 \mathrm{kN})(\text { does not govern })
\end{aligned}
$$

Reinforcement for confinement $=\rho_{\mathrm{s}}=0.00623 \therefore$ Provide \#4 at 2.9 in . (\#15 at 74 mm )

## Summary of design:

$4 \mathrm{ft}(1.22 \mathrm{~m})$ diameter of column with 26-\#9 (26-\#30) for main reinforcement and \#4 at 2.9 in . (\#15 at 74 mm ) for spiral confinement.


FIGURE 2.11 Typical cross sections of composite columns.

### 2.4.4 Steel and Composite Columns

Steel columns are not as commonly used as concrete columns. Nevertheless, they are viable solutions for some special occasions, e.g., in space-restricted areas. Steel pipes or tubes filled with concrete known as composite columns (Figure 2.11) offer the most efficient use of the two basic materials. Steel at the perimeter of the cross section provides stiffness and triaxial confinement, and the concrete core resists compression and prohibits local elastic buckling of the steel encasement. The toughness and ductility of composite columns makes them the preferred column type for earth-quake-resistant structures in Japan. In China, the composite columns were first used in Beijing subway stations as early as 1963 . Over the years, the composite columns have been used extensively in building structures as well as in bridges [6-9].

In this section, the design provisions of AASHTO LRFD [1] for steel and composite columns are summarized.

## Compressive Resistance

For prismatic members with at least one plane of symmetry and subjected to either axial compression or combined axial compression and flexure about an axis of symmetry, the factored resistance of components in compression, $P_{r}$, is calculated as

$$
P_{r}=\phi_{c} P_{n}
$$

where
$P_{n}=$ nominal compressive resistance
$\phi_{c}=$ resistance factor for compression $=0.90$
The nominal compressive resistance of a steel or composite column should be determined as

$$
P_{n}=\left\{\begin{array}{lll}
0.66^{\lambda} F_{e} A_{s} & \text { if } & \lambda \leq 2.25  \tag{2.26}\\
\frac{0.88 F_{e} A_{s}}{\lambda} & \text { if } & \lambda>2.25
\end{array}\right.
$$

in which
For steel columns:

$$
\begin{equation*}
\lambda=\left(\frac{K L}{r_{s}} \pi\right)^{2} \frac{F_{y}}{E_{e}} \tag{2.27}
\end{equation*}
$$

For composite column:

$$
\begin{gather*}
\lambda=\left(\frac{K L}{r_{s}} \pi\right)^{2} \frac{F_{e}}{E_{e}}  \tag{2.28}\\
F_{e}=F_{y}+C_{1} F_{y r}\left(\frac{A_{r}}{A_{s}}\right)+C_{2} f_{c}\left(\frac{A_{c}}{A_{s}}\right)  \tag{2.29}\\
E_{e}=E\left[1+\left(\frac{C_{3}}{n}\right)\left(\frac{A_{c}}{A_{s}}\right)\right] \tag{2.30}
\end{gather*}
$$

where
$A_{s}=$ cross-sectional area of the steel section $\left(\mathrm{mm}^{2}\right)$
$A_{c}=$ cross-sectional area of the concrete ( $\mathrm{mm}^{2}$ )
$A_{r}=$ total cross-sectional area of the longitudinal reinforcement ( $\mathrm{mm}^{2}$ )
$F_{y}=$ specified minimum yield strength of steel section (MPa)
$F_{y r}=$ specified minimum yield strength of the longitudinal reinforcement (MPa)
$f_{c}^{\prime}=$ specified minimum 28-day compressive strength of the concrete (MPa)
$E=$ modules of elasticity of the steel (MPa)
$L=$ unbraced length of the column (mm)
$K=$ effective length factor
$n=$ modular ratio of the concrete
$r_{s}=$ radius of gyration of the steel section in the plane of bending, but not less than 0.3 times the width of the composite member in the plane of bending for composite columns, and, for filled tubes,

$$
C_{1}=1.0 ; \quad C_{2}=0.85 ; \quad C_{3}=0.40
$$

In order to use the above equation, the following limiting width/thickness ratios for axial compression of steel members of any shape must be satisfied:

$$
\begin{equation*}
\frac{b}{t} \leq k \sqrt{\frac{E}{F_{y}}} \tag{2.31}
\end{equation*}
$$

where
$k=$ plate buckling coefficient as specified in Table 2.7
$b=$ width of plate as specified in Table 2.7
$t=$ plate thickness (mm)
Wall thickness of steel or composite tubes should satisfy:
For circular tubes:

$$
\frac{D}{t} \leq 2.8 \sqrt{\frac{E}{F_{y}}}
$$

TABLE 2.7 Limiting Width-to-Thickness Ratios

|  | k | Plates Supported along One Edge |
| :--- | :---: | :--- |
| Flanges and projecting <br> leg or plates | 0.56 | Half-flange width of I-section <br> Full-flange width of channels <br> Distance between free edge and first line of bolts or welds in plates <br> Full-width of an outstanding leg for pairs of angles on continuous contact |
| Stems of rolled tees |  |  |
| Other projecting elements | 0.75 <br> Full-depth of tee <br> Full-width of outstanding leg for single-angle strut or double-angle strut with <br> separator |  |
| Full projecting width for others |  |  |

## For rectangular tubes:

$$
\frac{b}{t} \leq 1.7 \sqrt{\frac{E}{F_{y}}}
$$

where
$D=$ diameter of tube (mm)
$b=$ width of face (mm)
$t=$ thickness of tube (mm)

## Flexural Resistance

The factored flexural resistance, $M_{p}$, should be determined as

$$
\begin{equation*}
M_{r}=\phi_{f} M_{n} \tag{2.32}
\end{equation*}
$$

where
$M_{n}=$ nominal flexural resistance
$\phi_{f}=$ resistance factor for flexure, $\phi_{f}=1.0$
The nominal flexural resistance of concrete-filled pipes that satisfy the limitation

$$
\frac{D}{t} \leq 2.8 \sqrt{\frac{E}{F_{y}}}
$$

may be determined:

$$
\begin{equation*}
\text { If } \frac{D}{t}<2.0 \sqrt{\frac{E}{F_{y}}} \text {, then } M_{n}=M_{p s} \tag{2.33}
\end{equation*}
$$

$$
\begin{equation*}
\text { If } 2.0 \sqrt{\frac{E}{F_{y}}}<\frac{D}{t} \leq 8.8 \sqrt{\frac{E}{F_{y}}} \text {, then } M_{n}=M_{y c} \tag{2.34}
\end{equation*}
$$

where
$M_{p s}=$ plastic moment of the steel section
$M_{y c}=$ yield moment of the composite section

## Combined Axial Compression and Flexure

The axial compressive load, $P_{u}$, and concurrent moments, $M_{u x}$ and $M_{u p}$, calculated for the factored loadings for both steel and composite columns should satisfy the following relationship:

$$
\begin{align*}
& \text { If } \frac{P_{u}}{P_{r}}<0.2, \text { then } \frac{P_{u}}{2.0 P_{r}}+\left(\frac{M_{u x}}{M_{r x}}+\frac{M_{u y}}{M_{r y}}\right) \leq 1.0  \tag{2.35}\\
& \text { If } \frac{P_{u}}{P_{r}} \geq 0.2 \text {, then } \frac{P_{u}}{P_{r}}+\frac{8.0}{9.0}\left(\frac{M_{u x}}{M_{r x}}+\frac{M_{u y}}{M_{r y}}\right) \leq 1.0 \tag{2.36}
\end{align*}
$$

where
$P_{r} \quad=$ factored compressive resistance
$M_{r x}, M_{r y}=$ factored flexural resistances about $x$ and $y$ axis, respectively
$M_{u x}, M_{u y}=$ factored flexural moments about the $x$ and $y$ axis, respectively

## References

1. AASHTO, LRFD Bridge Design Specifications, 1st ed., American Association of State Highway and Transportation Officials, Washington, D.C., 1994.
2. Caltrans, Bridge Memo to Designers (7-10), California Department of Transportation, Sacramento, 1994.
3. White, D. W. and Hajjar, J. F., Application of second-order elastic analysis in LRFD: research to practice, Eng. J., 28(4), 133, 1994.
4. Galambos, T. V., Ed., Guide to Stability Design for Metal Structures, 4th ed., the Structural Stability Research Council, John Wiley \& Sons, New York, 1988.
5. ATC, Improved Seismic Design Criteria for California Bridges: Provisional Recommendations, Applied Technology Council, Report ATC-32, Redwood City, CA, 1996.
6. Cai, S.-H., Chinese standard for concrete-filled tube columns, in Composite Construction in Steel and Concrete II, Proc. of an Engineering Foundation Conference, Samuel Easterling, W. and Kim Roddis, W. M., Eds, Potosi, MO, 1992, 143.
7. Cai, S.-H., Ultimate strength of concrete-filled tube columns, in Composite Construction in Steel and Concrete, Proc. of an Engineering Foundation Conference, Dale Buckner, C. and Viest, I. M., Eds, Henniker, NH, 1987, 703.
8. Zhong, S.-T., New concept and development of research on concrete-filled steel tube (CFST) members, in Proc. 2nd Int. Symp. on Civil Infrastructure Systems, 1996.
9. CECS 28:90, Specifications for the Design and Construction of Concrete-Filled Steel Tubular Structures, China Planning Press, Beijing [in Chinese], 1990.
10. AISC, Load and Research Factor Design Specification for Structural Steel Buildings and Commentary, 2nd ed., American Institute of Steel Construction, Chicago, IL, 1993.
11. Galambos, T. V. and Chapuis, J., LRFD Criteria for Composite Columns and Beam Columns, Revised Draft, Washington University, Department of Civil Engineering, St. Louis, MO, December 1990.
